

Complete type IIA superstring action on IIA plane wave background

Jaemo Park^{a*} and Hyeonjoon Shin^{b†}

^a*Department of Physics & Center for Theoretical Physics (PCTP),
POSTECH, Pohang 790-784, South Korea*

^b*Department of Physics, Pohang University of Science and Technology (POSTECH),
and Asia Pacific Center for Theoretical Physics (APCTP),
Pohang 790-784, South Korea*

Abstract

We construct the type IIA Green-Schwarz superstring action on a ten-dimensional IIA plane wave background with 24 supersymmetries keeping the full 32 fermionic coordinates. Starting from the symmetry superalgebra for the maximally supersymmetric eleven dimensional plane wave background, we obtain the eleven dimensional superfields. The Kaluza-Klein reduction leads to the ten dimensional superfields for the IIA plane wave background, from which the type IIA superstring action is constructed. We show that the superstring action reduces correctly to the previously known light-cone gauge fixed action upon imposing the light-cone κ -symmetry fixing condition.

Keywords : superstring action, plane wave, Kaluza-Klein reduction

*jaemo@postech.ac.kr

†hyeonjoon@postech.ac.kr

1 Introduction

Recent years have seen the tremendous development in our understanding of $\text{AdS}_4/\text{CFT}_3$ correspondence. Important inflection point is proposal by J. Schwarz [1] that underlying CFT_3 can be written as Chern-Simons matter (CSM) theories without the usual kinetic term for the gauge fields, especially for higher supersymmetric theories with $\mathcal{N} \geq 4$. Once that proposal is realized in specific examples, the understanding of $\text{AdS}_4/\text{CFT}_3$ correspondence has grown by leaps and bounds. After the realization that the Bagger-Lambert-Gustavsson theory [2–6] can be written as the usual $SU(2) \times SU(2)$ Chern-Simons matter theory [7], there appeared a paper by Gaiotto and Witten [8] where the attempt is made to write down $\mathcal{N} = 4$ Chern-Simons matter theory with matter hypermultiplets. The attempt was generalized in [9] which includes twisted hypermultiplets as well, thereby writing down the general classes of $\mathcal{N} = 4$ Chern-Simons matter theories. The special case of such construction is the famous $\mathcal{N} = 6$ theory, known as ABJM theory [10,11], describing coincident M2 branes on C^4/\mathbf{Z}_k where $(k, -k)$ is the Chern-Simons level for two gauge groups of ABJM theory. If the level k is taken to be infinite, the correspondence becomes that between the ABJM theory and type IIA superstring theory on $\text{AdS}_4 \times CP^3$.

For the further study of the $\text{AdS}_4/\text{CFT}_3$ correspondence, the first step in the bulk side is to have the IIA superstring action on $\text{AdS}_4 \times CP^3$, which has been constructed based on the super coset formulation in [12]. However, as pointed out already in the coset formulation and emphasized in a subsequent work [13], the constructed superstring action contains only 24 fermionic components although type IIA superstring has 32 fermionic components, and thus is not complete. This problem has been cured by incorporating the missing 8 fermionic components through a suitable extension of the coset superspace, and the complete type IIA superstring action on $\text{AdS}_4 \times CP^3$ has been constructed in [13].

The complete type IIA superstring action opens up the way of investigating various possible string configurations. However, it has rather complicated structure, which may make the full understanding of string or the quantization of string on the $\text{AdS}_4 \times CP^3$ background not so easy. In this case, some simple background would be helpful. One typical and successful example of such simplification may be the Penrose limit of the $\text{AdS}_5 \times S^5$ background which leads to the IIB plane wave background [14,15]. The type IIB superstring action constructed on this IIB plane wave background has turned out to be exactly solvable [16], and triggered the significant progress in the study of $\text{AdS}_5/\text{CFT}_4$ correspondence [17]. One may expect that a similar simplified theory can be constructed also in the present case. Indeed, it has been shown in [18–20] that a suitable Penrose limit of the $\text{AdS}_4 \times CP^3$

background leads to the previously known IIA plane wave background [21–23]. Similar to the IIB case, the type IIA superstring theory on this background is also solvable and can be quantized [24]. This manageable situation through the simplification has been exploited in various works on the study of $\text{AdS}_4/\text{CFT}_3$ correspondence [25–30]. We expect this type of investigation would lead to better understanding of the stringy aspects of $\text{AdS}_4/\text{CFT}_3$ correspondence.

In this paper, we revisit the type IIA Green-Schwarz superstring action on the IIA plane wave background and try to construct the complete action containing all the 32 fermionic components. Basically, the motivation comes from the fact that the IIA plane wave background is not maximally supersymmetric. As pointed out in [13] and explored in more detail in [31], when the superstring action on a less supersymmetric background is considered, the κ -symmetry fixing condition should be chosen carefully in studying a particular motion of string. For example, the type IIA superstring action on $\text{AdS}_4 \times CP^3$ constructed based on the super coset does not give the correct description of string moving only in AdS_4 space, because it fixes from the beginning 8 fermionic components corresponding to the broken supersymmetries though those components play important role in describing such string motion. In this sense, the complete action containing all the 32 fermionic components is required for studying various possible string configurations or motions.

In the previous construction of type IIA superstring action on IIA plane wave background [21, 22], the Penrose limit was taken for the membrane action on $\text{AdS}_4 \times S^7$ [32], and the double dimensional reduction was performed after taking the light-cone gauge fixing condition in eleven dimensions for simplicity. We could take the same steps for obtaining the complete action without any gauge fixing. In this paper, however, we will take a different route. Starting from the symmetry superalgebra of eleven dimensional plane wave background, the eleven dimensional superfields will be obtained by following the recipe conceived in [33]. Then, through the Kaluza-Klein (KK) reduction, we will derive the ten dimensional superfields which are necessary in constructing the superstring action. In this way, we obtain another example of the Green-Schwarz superstring action with κ -symmetry for a background with non-maximal supersymmetry, whose supergravity constraints can be completely solved.

The organization of this paper is as follows. In the next section, we review the approach of [33], where one can read off the 10d/11d superfields for a background described by superalgebra and the associated coset structure. In section 3, the superfields for the eleven dimensional plane wave background are obtained starting from the symmetry superalgebra of the background. The KK reduction is performed in Sec. 4, which leads to the ten dimen-

sional superfields. In Sec. 5, the complete type IIA superstring action on IIA plane wave background is written down. As a check, we show that the action reduces correctly to the light-cone gauge fixed action constructed previously. Sec. 6 is devoted to conclusion. Our notation is summarized in an appendix.

2 Preliminary

If underlying geometry can be described by superalgebra and associated supercoset structure, the supergravity constraints can be completely solved. We review this approach by closely following [33] and apply it to 11-dimensional plane wave background in Sec. 3.

The general superalgebra is of the form

$$\begin{aligned} [B_{\hat{r}}, B_{\hat{s}}] &= f_{\hat{r}\hat{s}}^{\hat{t}} B_{\hat{t}} \\ \{F_{\hat{a}}, B_{\hat{r}}\} &= f_{\hat{a}\hat{r}}^{\hat{b}} F_{\hat{b}} \\ \{F_{\hat{a}}, F_{\hat{b}}\} &= f_{\hat{a}\hat{b}}^{\hat{r}} F_{\hat{r}} \end{aligned} \quad (2.1)$$

where $B_{\hat{r}}$ and $F_{\hat{a}}$ are the bosonic and fermionic generators, respectively and f are the structure constants. The differential operator is defined to be

$$D = d + L^{\hat{r}} B_{\hat{r}} + L^{\hat{a}} B_{\hat{a}} \quad (2.2)$$

where $L^{\hat{r}}, L^{\hat{a}}$ are the left-invariant Cartan one-forms. The equation D^2 leads to the usual Maurer-Cartan equation

$$\begin{aligned} dL^{\hat{r}} + \frac{1}{2} f_{\hat{s}\hat{t}}^{\hat{r}} L^{\hat{s}} \wedge L^{\hat{t}} - \frac{1}{2} f_{\hat{a}\hat{b}}^{\hat{r}} L^{\hat{a}} \wedge L^{\hat{b}} &= 0 \\ dL^{\hat{a}} + f_{\hat{r}\hat{t}}^{\hat{a}} L^{\hat{r}} \wedge L^{\hat{t}} &= 0 \end{aligned} \quad (2.3)$$

If the underlying space has the supercoset structure, the Maurer-Cartan equation can be solved completely. Writing

$$G = g(x) e^{\theta F}, \quad (2.4)$$

we find that

$$G^{-1} dG = e^{-\theta F} D e^{\theta F} \quad (2.5)$$

with $D = d + L_0^{\hat{a}} B_{\hat{a}}$. Here $L_0^{\hat{a}}$ is defined as

$$L^{\hat{A}} = L_0^{\hat{A}}(x) + \tilde{L}^{\hat{A}}(x, \theta) \quad (2.6)$$

with $\hat{A} = (\hat{r}, \hat{a})$ collectively. If one introduces λ dependence by $\theta \rightarrow \lambda\theta$,

$$e^{-\lambda\theta} d e^{\lambda\theta} = \tilde{L}_{\lambda}^{\hat{r}} B_{\hat{r}} + \tilde{L}_{\lambda}^{\hat{a}} F_{\hat{a}}. \quad (2.7)$$

By differentiating both sides and using the superalgebra, we obtain

$$\begin{aligned}\partial_\lambda \tilde{L}_\lambda^{\hat{r}} &= \theta^{\hat{a}} \tilde{L}_\lambda^{\hat{b}} f_{\hat{a}\hat{b}}^{\hat{r}} \\ \partial_\lambda \tilde{L}_\lambda^{\hat{a}} &= d\theta^{\hat{a}} - \theta^{\hat{b}} f_{\hat{b}\hat{r}}^{\hat{a}} \tilde{L}_\lambda^{\hat{r}}.\end{aligned}\tag{2.8}$$

With the initial condition $\tilde{L}_{\lambda=0}^{\hat{r}} = \tilde{L}_{\lambda=0}^{\hat{a}} = 0$, these equations can be solved

$$\begin{aligned}L^{\hat{a}} &= \left(\frac{\sinh \mathcal{M}}{\mathcal{M}} \right)^{\hat{a}}_{\hat{b}} d\theta^{\hat{b}} \\ L^{\hat{r}} &= L_0^{\hat{r}} + 2\theta^{\hat{a}} f_{\hat{a}\hat{b}}^{\hat{r}} \left(\frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \right)^{\hat{b}}_{\hat{r}} d\theta^{\hat{r}},\end{aligned}\tag{2.9}$$

where $(\mathcal{M}^2)^{\hat{a}}_{\hat{b}} = -\theta^{\hat{c}} f_{\hat{c}\hat{r}}^{\hat{a}} \theta^{\hat{d}} f_{\hat{d}\hat{b}}^{\hat{r}}$. If we choose θ to be the standard fermion coordinates of the superspace, this gives the superspace geometry in Wess-Zumino gauge.

If the above background describes the 11-d background, the membrane action can be written as

$$S = \int d^3\xi \sqrt{-\det L_i^{\hat{r}} L_j^{\hat{s}} \eta_{\hat{r}\hat{s}}} - \frac{1}{6} L_i^{\hat{A}} L_j^{\hat{B}} L_k^{\hat{C}} B_{\hat{A}\hat{B}\hat{C}}\tag{2.10}$$

where $B_{\hat{A}\hat{B}\hat{C}}$ represents 3-form superfield, which should be determined separately. Later we will do this for 11-d plane-wave background in section 3.

Once we obtain the membrane action in 11-d then we carry out the double dimensional reduction to obtain the string action in 10-d plane wave background.

3 Superfields for the eleven dimensional plane wave background

3.1 Symmetry superalgebra

The eleven dimensional plane wave background [34] is one of the maximally supersymmetric solutions in eleven dimensional supergravity [35] and is given by

$$\begin{aligned}ds^2 &= 2dx^+ dx^- - \left(\sum_{\hat{i}=1}^3 \frac{\mu^2}{9} (x^{\hat{i}})^2 + \sum_{\hat{i}'=4}^9 \frac{\mu^2}{36} (x^{\hat{i}'})^2 \right) (dx^-)^2 + (dx^{\hat{I}})^2, \\ F_{-123} &= \mu,\end{aligned}\tag{3.1}$$

where $x^\pm = \frac{1}{\sqrt{2}}(x^{11} \pm x^0)$ and $\hat{I} = (\hat{i}, \hat{i}')$. Apart from an interesting solution, this background has an important connection with the AdS type backgrounds such as $\text{AdS}_4 \times S^7$ or $\text{AdS}_7 \times S^4$ through the Penrose limit [15]. In this case, the dimensionful parameter μ is inversely proportional to the radius R of the AdS space, $\mu \propto 1/R$.

More detailed information on the plane wave background (3.1) is given by the underlying superalgebra, which has been obtained from the investigation of isometries in [36]. Regarding the relation between the plane wave and the AdS type backgrounds, it has been shown that the same symmetry superalgebra can be derived also from the superalgebra of $\text{AdS}_4 \times S^7$ or $\text{AdS}_7 \times S^4$ via the Inönü-Wigner contraction which can be regarded as the algebraic version of the Penrose limit [37]. Referring to Refs. [36, 37], the superalgebra is given as follows.¹ Firstly, the commutation relations between the bosonic generators are

$$\begin{aligned} [P_{\hat{I}}, P_-] &= -P_{\hat{I}}^*, \quad [P_{\hat{I}}^*, P_-] = \frac{\mu^2}{9} P_{\hat{I}}, \quad [P_{\hat{i}'}^*, P_-] = \frac{\mu^2}{36} P_{\hat{i}'}, \\ [P_{\hat{i}}^*, P_{\hat{j}}] &= -\frac{\mu^2}{9} \eta_{\hat{i}\hat{j}} P_+, \quad [P_{\hat{i}'}^*, P_{\hat{j}'}] = -\frac{\mu^2}{36} \eta_{\hat{i}'\hat{j}'} P_+, \\ [J_{\hat{I}\hat{J}}, P_{\hat{K}}] &= 2\eta_{\hat{J}\hat{K}} P_{\hat{I}}, \quad [J_{\hat{I}\hat{J}}, P_{\hat{K}}^*] = 2\eta_{\hat{J}\hat{K}} P_{\hat{I}}^*, \quad [J_{\hat{I}\hat{J}}, J_{\hat{K}\hat{L}}] = 4\eta_{\hat{J}\hat{K}} J_{\hat{I}\hat{L}}, \end{aligned} \quad (3.2)$$

where $\hat{i} = 1 \cdots 3, \hat{i}' = 4 \cdots 9$ and \hat{I}, \hat{J} denote $SO(9)$ vector index. The details of the convention are summarized at the appendix. Also P and J denote the translation and the rotation generators respectively and

$$P_{\pm} \equiv \frac{1}{\sqrt{2}}(P_{11} \pm P_0), \quad P_{\hat{i}}^* \equiv J_{\hat{i}0}, \quad P_{\hat{i}'}^* \equiv J_{\hat{i}'11}. \quad (3.3)$$

Secondly, the algebra between the bosonic and the fermionic generators is²

$$\begin{aligned} [P_-, Q_+] &= -\frac{\mu}{4} Q_+ \Pi, \quad [P_-, Q_-] = -\frac{\mu}{12} Q_- \Pi, \quad [P_{\hat{i}}, Q_-] = \frac{\mu}{6} Q_+ \Gamma^- \Pi \Gamma_{\hat{i}}, \\ [P_{\hat{i}'}, Q_-] &= \frac{\mu}{12} Q_+ \Gamma^- \Pi \Gamma_{\hat{i}'}, \quad [P_{\hat{i}}^*, Q_-] = \frac{\mu^2}{18} Q_+ \Gamma_{\hat{i}} \Gamma^-, \quad [P_{\hat{i}'}^*, Q_-] = \frac{\mu^2}{72} Q_+ \Gamma_{\hat{i}'} \Gamma^-, \end{aligned} \quad (3.4)$$

where Γ 's are 32×32 Dirac gamma matrices,

$$\Pi \equiv \Gamma^{123}, \quad (3.5)$$

and the supersymmetry generator Q with 32 components has been split into two parts by introducing a projection operator \mathcal{P}_{\pm} as

$$\mathcal{P}_{\pm} \equiv \frac{1}{2} \Gamma_{\pm} \Gamma_{\mp}, \quad Q_{\pm} \equiv Q \mathcal{P}_{\pm}, \quad Q = Q_+ + Q_-. \quad (3.6)$$

Finally, the algebra of supercharges is

$$\{Q_+, Q_+\} = -2C\Gamma^+ P_+,$$

¹We follow the algebra given in [36] but with the notation of [37] for generators. We also take a rescaling of supercharge as $Q \rightarrow Q/\sqrt{2}$ for our convenience.

²Throughout the paper, we suppress the spinor indices unless there is some confusion.

$$\begin{aligned}
\{Q_-, Q_-\} &= -2C\Gamma^- P_- - \frac{\mu}{3}C\Gamma^- \Pi\Gamma^{\hat{i}\hat{j}} J_{\hat{i}\hat{j}} + \frac{\mu}{6}C\Gamma^- \Pi\Gamma^{\hat{i}'\hat{j}'} J_{\hat{i}'\hat{j}'}, \\
\{Q_+, Q_-\} &= -2C\Gamma^{\hat{I}} P_{\hat{I}} - \frac{6}{\mu}C\Pi\Gamma^{\hat{i}} P_{\hat{i}}^* - \frac{12}{\mu}C\Pi\Gamma^{\hat{i}'} P_{\hat{i}'}^*,
\end{aligned} \tag{3.7}$$

where C is the charge conjugation matrix satisfying $C\Gamma^{\hat{r}}C^{-1} = -\Gamma^{\hat{r}T}$. Here, it seems that the last anticommutation relation is problematic in the limit of $\mu \rightarrow 0$, which is nothing but the flat spacetime limit as can be seen from the background (3.1). Although it should be reduced to the superalgebra of flat spacetime in this limit, it is apparently divergent. However this is just an artifact of convention. Indeed, as pointed out explicitly in [37], a proper rescaling of superalgebra generators cures the problem of flat spacetime limit.

3.2 Cartan one-forms

The symmetry superalgebra, (3.2), (3.4), and (3.7), is the full superalgebra of the plane wave background (3.1) and one feature of it is that it has the form of the algebraic structure of coset superspace \mathcal{G}/\mathcal{H} [36]. (The whole generators are those of the group \mathcal{G} and the rotation generators correspond to those of the stability subgroup \mathcal{H} .) This implies that, by starting from the full superalgebra and following the general prescription suggested in [33, 38], we can read off the superspace geometry represented by superfields which is our goal in this section.

In the case where the superspace has the coset structure, the superfields are expressed in terms of the left-invariant Cartan one-forms. If we let G be the coset representative, then

$$G^{-1}dG = L^{\hat{r}}P_{\hat{r}} + L_{*}^{\hat{i}}P_{\hat{i}}^* + L_{*}^{\hat{i}'}P_{\hat{i}'}^* + \frac{1}{2}L^{\hat{I}\hat{J}}J_{\hat{I}\hat{J}} + \mathbf{L}^+Q_+ + \mathbf{L}^-Q_- \tag{3.8}$$

gives the left-invariant Cartan one-forms, $L^{\hat{r}}$, $L_{*}^{\hat{i}}$, $L_{*}^{\hat{i}'}$, $L^{\hat{I}\hat{J}}$, \mathbf{L}^+ , and \mathbf{L}^- , where the spinorial one-forms \mathbf{L}^{\pm} are the projections of the spinorial one-form L just like Q_{\pm} of Eq. (3.6);

$$\mathbf{L}^{\pm} \equiv \mathcal{P}_{\pm}L, \quad L = \mathbf{L}^+ + \mathbf{L}^-. \tag{3.9}$$

We note that $L^{\hat{i}\hat{j}'} = 0$ because there is no generator $J_{\hat{i}\hat{j}'}$ in the superalgebra of (3.2), (3.4), and (3.7). Now, from the integrability of (3.8) and the superalgebra, the left-invariant Cartan one-forms turn out to satisfy the Maurer-Cartan equations,

$$\begin{aligned}
dL^+ - \frac{\mu^2}{9}L_{*}^{\hat{i}} \wedge L^{\hat{i}} - \frac{\mu^2}{36}L_{*}^{\hat{i}'} \wedge L^{\hat{i}'} + \bar{L} \wedge \Gamma^+ L &= 0, \\
dL^- + \bar{L} \wedge \Gamma^- L &= 0, \\
dL^{\hat{i}} + \frac{\mu^2}{9}L_{*}^{\hat{i}} \wedge L^- + L^{\hat{i}\hat{j}} \wedge L^{\hat{j}} + \bar{L} \wedge \Gamma^{\hat{i}} L &= 0, \\
dL^{\hat{i}'} + \frac{\mu^2}{36}L_{*}^{\hat{i}'} \wedge L^- + L^{\hat{i}'\hat{j}'} \wedge L^{\hat{j}'} + \bar{L} \wedge \Gamma^{\hat{i}'} L &= 0,
\end{aligned}$$

$$\begin{aligned}
dL_*^{\hat{i}} - L^{\hat{i}} \wedge L^- + L^{\hat{i}\hat{j}} \wedge L_*^{\hat{j}} + \frac{3}{\mu} \bar{L} \wedge \Pi \Gamma^{\hat{i}} L &= 0, \\
dL_*^{\hat{i}'} - L^{\hat{i}'} \wedge L^- + L^{\hat{i}'\hat{j}'} \wedge L_*^{\hat{j}'} + \frac{6}{\mu} \bar{L} \wedge \Pi \Gamma^{\hat{i}'} L &= 0, \\
dL^{\hat{i}\hat{j}} + L^{\hat{k}\hat{i}} \wedge L^{\hat{j}\hat{k}} + \frac{\mu}{6} \bar{L} \wedge \Gamma^- \Pi \Gamma^{\hat{i}\hat{j}} L &= 0, \\
dL^{\hat{i}'\hat{j}'} + L^{\hat{k}'\hat{i}'} \wedge L^{\hat{j}'\hat{k}'} + \frac{\mu}{12} \bar{L} \wedge \Gamma^- \Pi \Gamma^{\hat{i}'\hat{j}'} L &= 0, \\
dL + \frac{1}{4} L^{\hat{i}\hat{j}} \wedge \Gamma_{\hat{i}\hat{j}} L - \frac{\mu}{12} L^- \wedge \Pi(\Gamma^- \Gamma^+ + 1)L + \frac{\mu}{6} L^{\hat{i}} \wedge \Gamma^- \Pi \Gamma_{\hat{i}} L \\
+ \frac{\mu}{12} L^{\hat{i}'} \wedge \Gamma^- \Pi \Gamma_{\hat{i}'} L + \frac{\mu^2}{18} L_*^{\hat{i}} \wedge \Gamma_{\hat{i}} \Gamma^- L + \frac{\mu^2}{72} L_*^{\hat{i}'} \wedge \Gamma_{\hat{i}'} \Gamma^- L &= 0, \tag{3.10}
\end{aligned}$$

where $\bar{L} = L^T C$.

3.3 Eleven dimensional superfields

The left-invariant Cartan one-forms are functions of the supercoordinate composed of $x^{\hat{r}}$ and $\theta^{\hat{a}}$ parametrizing the superspace, and each of them has its own expansion in terms of θ . Since their expanded form is eventually necessary in the construction of the superstring action, we now determine them to all orders in θ . In order to do this, we begin with making a particular choice of the coset representative G in (3.8) as

$$G(x, \theta) = g(x) e^{\theta^+ Q_+ + \theta^- Q_-}, \tag{3.11}$$

which is known as the Wess-Zumino type parametrization. The bosonic factor $g(x)$ is for the purely bosonic part of the superspace and is left unspecified. The fermionic coordinates in the exponential factor are defined by, using the projection operator in (3.6),

$$\theta^{\pm} \equiv \mathcal{P}_{\pm} \theta, \quad \theta = \theta^+ + \theta^-. \tag{3.12}$$

As a next step, we take the rescaling $\theta \rightarrow \lambda \theta$ with an auxiliary parameter λ [33, 38] and put a subscript λ for rescaled quantities such as, for example, $G_{\lambda} = G(x, \lambda \theta)$ and $L_{\lambda}^{\hat{r}} = L^{\hat{r}}(x, \lambda \theta)$. Then the differentiation with respect to λ of (3.8) with (3.11) and the superalgebra, (3.2), (3.4), and (3.7), lead us to have

$$\begin{aligned}
\partial_{\lambda} L_{\lambda}^{\hat{r}} &= 2 \bar{L}_{\lambda} \Gamma^{\hat{r}} \theta, \\
\partial_{\lambda} L_{*\lambda}^{\hat{i}} &= \frac{6}{\mu} \bar{L}_{\lambda} \Pi \Gamma^{\hat{i}} \theta, \quad \partial_{\lambda} L_{*\lambda}^{\hat{i}'} = \frac{12}{\mu} \bar{L}_{\lambda} \Pi \Gamma^{\hat{i}'} \theta, \\
\partial_{\lambda} L_{\lambda}^{\hat{i}\hat{j}} &= \frac{\mu}{3} \bar{L}_{\lambda} \Gamma_+ \Pi \Gamma^{\hat{i}\hat{j}} \theta, \quad \partial_{\lambda} L_{\lambda}^{\hat{i}'\hat{j}'} = -\frac{\mu}{6} \bar{L}_{\lambda} \Gamma_+ \Pi \Gamma^{\hat{i}'\hat{j}'} \theta, \\
\partial_{\lambda} L_{\lambda} &= d\theta + \frac{1}{4} L_{\lambda}^{\hat{i}\hat{j}} \Gamma_{\hat{i}\hat{j}} \theta - \frac{\mu}{12} L_{\lambda}^- \Pi(\Gamma^- \Gamma^+ + 1) \theta
\end{aligned}$$

$$+ \frac{\mu}{6} \left(L_{\lambda}^{\hat{i}} \Gamma^{-} \Pi \Gamma_{\hat{i}} + \frac{1}{2} L_{\lambda}^{\hat{i}'} \Gamma^{-} \Pi \Gamma_{\hat{i}'} + \frac{\mu}{3} L_{*\lambda}^{\hat{i}} \Gamma_{\hat{i}} \Gamma^{-} + \frac{\mu}{12} L_{*\lambda}^{\hat{i}'} \Gamma_{\hat{i}'} \Gamma^{-} \right) \theta. \quad (3.13)$$

As pointed out in [33], these first-order differential equations have the structure of coupled harmonic oscillators, and thus can be solved exactly. To solve these equations, we first impose the initial conditions for the Cartan one-forms as

$$L_{\lambda=0}^{\hat{r}} = \hat{e}^{\hat{r}}, \quad L_{\lambda=0}^{\hat{i}\hat{j}} = \hat{\omega}^{\hat{i}\hat{j}}, \quad L_{\lambda=0}^{\hat{i}'\hat{j}'} = \hat{\omega}^{\hat{i}'\hat{j}'}, \quad L_{\lambda=0} = 0, \quad (3.14)$$

where $\hat{e}^{\hat{r}}$ is the elfbein and $\hat{\omega}^{\hat{i}\hat{j}}, \hat{\omega}^{\hat{i}'\hat{j}'}$ are the spin connections of the plane wave geometry. We note that $\hat{\omega}^{\hat{i}\hat{j}'} = 0$ automatically due to the fact that $L^{\hat{i}\hat{j}'} = 0$ as alluded to previously. As for the remaining Cartan one-forms, one may be tempted to take $L_{*\lambda=0}^{\hat{i}} = \hat{\omega}^{\hat{i}0}$ and $L_{*\lambda=0}^{\hat{i}'} = \hat{\omega}^{\hat{i}'11}$ from the definition of (3.3). However, this is naive expectation. Indeed, if we compare the purely bosonic part of the Maurer-Cartan equation (3.10) with the usual Cartan structure equation, we can see that the correct initial conditions are

$$L_{*\lambda=0}^{\hat{i}} = -\frac{9}{\mu^2} \hat{\omega}^{+\hat{i}}, \quad L_{*\lambda=0}^{\hat{i}'} = -\frac{36}{\mu^2} \hat{\omega}^{+\hat{i}'}, \quad (3.15)$$

together with a consistency condition $\hat{\omega}^{-\hat{r}} = 0$. The initial conditions, (3.14) and (3.15), form an enough set of data for solving the differential equations of (3.13). Furthermore, they give us an information about non-vanishing derivative at $\lambda = 0$ [32], which is identified with the eleven dimensional covariant derivative for the fermionic coordinate denoted by $\hat{D}\theta$;

$$\hat{D}\theta \equiv \partial_{\lambda} L_{\lambda}|_{\lambda=0}. \quad (3.16)$$

If we consider the covariant derivatives for θ^{+} and θ^{-} separately, their explicit expressions are obtained as

$$\begin{aligned} \hat{D}\theta^{+} &= d\theta^{+} + \frac{1}{4} \hat{\omega}^{\hat{I}\hat{J}} \Gamma_{\hat{I}\hat{J}} \theta^{+} + \frac{1}{2} \hat{\omega}^{+\hat{I}} \Gamma_{+\hat{I}} \theta^{-} - \frac{\mu}{4} \hat{e}^{-} \Pi \theta^{+} \\ &\quad + \frac{\mu}{6} \left(\hat{e}^{\hat{i}} \Gamma^{-} \Pi \Gamma_{\hat{i}} + \frac{1}{2} \hat{e}^{\hat{i}'} \Gamma^{-} \Pi \Gamma_{\hat{i}'} \right) \theta^{-}, \\ \hat{D}\theta^{-} &= d\theta^{-} + \frac{1}{4} \hat{\omega}^{\hat{I}\hat{J}} \Gamma_{\hat{I}\hat{J}} \theta^{-} - \frac{\mu}{12} \hat{e}^{-} \Pi \theta^{-}. \end{aligned} \quad (3.17)$$

Now it is straightforward to solve the equations (3.13) with the initial conditions (3.14) and (3.15). After setting $\lambda = 1$, that is, $L = L_{\lambda=1}$, we finally have

$$L^{\hat{r}} = \hat{e}^{\hat{r}} - 2 \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta} \Gamma^{\hat{r}} \mathcal{M}^{2n} \hat{D}\theta,$$

$$\begin{aligned}
L_*^{\hat{i}} &= -\frac{9}{\mu^2}\hat{\omega}^{+\hat{i}} - \frac{6}{\mu}\sum_{n=0}^{15}\frac{1}{(2n+2)!}\bar{\theta}\Pi\Gamma^{\hat{i}}\mathcal{M}^{2n}\hat{D}\theta, \\
L_*^{\hat{i}'} &= -\frac{36}{\mu^2}\hat{\omega}^{+\hat{i}'} - \frac{12}{\mu}\sum_{n=0}^{15}\frac{1}{(2n+2)!}\bar{\theta}\Pi\Gamma^{\hat{i}'}\mathcal{M}^{2n}\hat{D}\theta, \\
L_*^{\hat{i}\hat{j}} &= \hat{\omega}^{\hat{i}\hat{j}} - \frac{\mu}{3}\sum_{n=0}^{15}\frac{1}{(2n+2)!}\bar{\theta}\Gamma^-\Pi\Gamma^{\hat{i}\hat{j}}\mathcal{M}^{2n}\hat{D}\theta, \\
L_*^{\hat{i}'\hat{j}'} &= \hat{\omega}^{\hat{i}'\hat{j}'} + \frac{\mu}{6}\sum_{n=0}^{15}\frac{1}{(2n+2)!}\bar{\theta}\Gamma^-\Pi\Gamma^{\hat{i}'\hat{j}'}\mathcal{M}^{2n}\hat{D}\theta, \\
L &= \sum_{n=0}^{16}\frac{1}{(2n+1)!}\mathcal{M}^{2n}\hat{D}\theta,
\end{aligned} \tag{3.18}$$

where $\hat{D}\theta = \hat{D}\theta^+ + \hat{D}\theta^-$ and \mathcal{M}^2 is the 32×32 matrix given by

$$\begin{aligned}
\mathcal{M}^2 &= \frac{\mu}{6}\left[(\Pi(\Gamma^-\Gamma^+ + 1)\theta)(\bar{\theta}\Gamma^-\mathcal{P}_-) - 2(\Gamma^-\Pi\Gamma_{\hat{i}}\mathcal{P}_-\theta)(\bar{\theta}\Gamma^{\hat{i}}) - (\Gamma^-\Pi\Gamma_{\hat{i}'}\mathcal{P}_-\theta)(\bar{\theta}\Gamma^{\hat{i}'}) \right. \\
&\quad - 2(\Gamma_{\hat{i}}\Gamma^-\mathcal{P}_-\theta)(\bar{\theta}\Pi\Gamma^{\hat{i}}) - (\Gamma_{\hat{i}'}\Gamma^-\mathcal{P}_-\theta)(\bar{\theta}\Pi\Gamma^{\hat{i}'}) - \frac{1}{2}(\Gamma_{\hat{i}\hat{j}}\theta)(\bar{\theta}\Gamma^-\Pi\Gamma^{\hat{i}\hat{j}}\mathcal{P}_-) \\
&\quad \left. + \frac{1}{4}(\Gamma_{\hat{i}'\hat{j}'}\theta)(\bar{\theta}\Gamma^-\Pi\Gamma^{\hat{i}'\hat{j}'}\mathcal{P}_-)\right].
\end{aligned} \tag{3.19}$$

The left-invariant Cartan one-forms of (3.18) are the superfields describing the superspace geometry. On the other hand, there is one more ingredient in the superspace for the eleven dimensional plane wave background. It is the three-form superfield \hat{B} , which forms the Wess-Zumino part of the supermembrane action. Basically, the problem is to find the closed four-form superfield H from a certain combination of various products of the superfields (3.18) and relate it to \hat{B} through the local equation $d\hat{B} = H$. In the present case, there are two possible candidates for H , which are $\bar{L}\wedge\Gamma_{\hat{r}\hat{s}}L\wedge L^{\hat{r}}\wedge L^{\hat{s}}$ and $L^{\hat{r}}\wedge L^{\hat{s}}\wedge L^{\hat{t}}\wedge L^{\hat{u}}F_{\hat{r}\hat{s}\hat{t}\hat{u}}$.³ We take a linear combination of these terms for H and fix the relative coefficient by requiring the closedness of H , that is, $dH = 0$. This process is performed by using the Maurer-Cartan equation (3.10) and the eleven dimensional Fierz identity $(C\Gamma_{\hat{r}\hat{s}})_{(\hat{a}\hat{b}}(C\Gamma^{\hat{s}})_{\hat{c}\hat{d}}) = 0$, and the resulting expression is obtained as

$$H = \frac{1}{4!}(L^{\hat{r}}\wedge L^{\hat{s}}\wedge L^{\hat{t}}\wedge L^{\hat{u}}F_{\hat{r}\hat{s}\hat{t}\hat{u}} + 12\bar{L}\wedge\Gamma_{\hat{r}\hat{s}}L\wedge L^{\hat{r}}\wedge L^{\hat{s}}), \tag{3.20}$$

³We note that two terms are the same with those for the supermembrane in $\text{AdS}_4 \times S^7$ or $\text{AdS}_7 \times S^4$ [32]. We may think that this is natural since the plane wave background is the Penrose limit of these two AdS type backgrounds and hence the formal structure of the Wess-Zumino part is expected to be unchanged although the details are different. For more comprehensive study on the Wess-Zumino part of the supermembrane as well as the super fivebrane in various backgrounds, see for example Ref. [39] where the systematic Chevalley-Eilenberg cohomology [40] has been used.

where the overall multiplicative constant has been fixed such that it leads to the standard form of the Wess-Zumino term, for example as in [32].

Having the expression of H , we first apply the trick of rescaling $\theta \rightarrow \lambda\theta$ to H ; $H_\lambda = H(x, \lambda\theta)$. Then the following identity provides the equation for finding \widehat{B} .

$$H_{\lambda=1} = H_{\lambda=0} + \int_0^1 d\lambda \partial_\lambda H_\lambda. \quad (3.21)$$

By using the differential equations (3.13), we find that

$$\partial_\lambda H_\lambda = d(\bar{\theta}\Gamma_{\hat{r}\hat{s}}L_\lambda \wedge L_\lambda^{\hat{r}} \wedge L_\lambda^{\hat{s}}). \quad (3.22)$$

If we plug this into (3.21) and use the local relation $d\widehat{B} = H = H_{\lambda=1}$, then we obtain

$$\widehat{B} = \frac{1}{6}\hat{e}^{\hat{r}} \wedge \hat{e}^{\hat{s}} \wedge \hat{e}^{\hat{t}} \widehat{C}_{\hat{r}\hat{s}\hat{t}} + \int_0^1 d\lambda \bar{\theta}\Gamma_{\hat{r}\hat{s}}L_\lambda \wedge L_\lambda^{\hat{r}} \wedge L_\lambda^{\hat{s}}, \quad (3.23)$$

where the first term on the right hand side has resulted from $H_{\lambda=0}$ and $\widehat{C}_{\hat{r}\hat{s}\hat{t}}$ is three-form gauge field whose field strength is $\widehat{F}_{\hat{r}\hat{s}\hat{t}\hat{u}} = 4\partial_{[\hat{r}}\widehat{C}_{\hat{s}\hat{t}\hat{u}]}$.

4 Kaluza-Klein reduction

Having the eleven dimensional superfields for the eleven dimensional plane wave background in hand, we carry out the KK reduction along a spatial isometry direction of the plane wave geometry and obtain the superfields in ten dimensions. Since the Cartan one-forms, $L^{\hat{r}}$ and L , and the three-form superfield \widehat{B} are the necessary elements for constructing the IIA superstring action, we will focus only on them.

4.1 Ten dimensional bosonic geometry

As we can see from the eleven dimensional plane wave geometry (3.1), there is no explicit spatial isometry direction. However, it has been shown [21, 22] that the geometry can be made to have such an isometry under a suitable coordinate transformation, which is taken by following the prescription suggested in [41]. In our convention, if x^9 is taken to be the desired isometry direction, the transformation is given by $x^+ \rightarrow x^+ - (\mu/6)x^4x^9$, $x^4 \rightarrow \cos(\mu x^-/6)x^4 - \sin(\mu x^-/6)x^9$, and $x^9 \rightarrow \sin(\mu x^-/6)x^4 + \cos(\mu x^-/6)x^9$ without changing other coordinates. From the transformed geometry, we may choose the elfbein as

$$\hat{e}^+ = dx^+ - \frac{1}{2}A(x^I)dx^-, \quad \hat{e}^- = dx^-, \quad \hat{e}^I = dx^I, \quad \hat{e}^9 = dx^9 + \frac{\mu}{3}x^4dx^-, \quad (4.1)$$

where

$$A(x^I) = \sum_{i=1}^4 \frac{\mu^2}{9} (x^i)^2 + \sum_{i'=5}^8 \frac{\mu^2}{36} (x^{i'})^2, \quad (4.2)$$

and $I = (i, i') = 1, \dots, 8$. Clearly, this choice has a suitable form for the KK reduction along x^9 basically because it satisfies the KK ansatz

$$\hat{e}_9^r = 0. \quad (4.3)$$

The above elfbein (4.1) is taken to be a parametrization of the purely bosonic part of the super elfbein, that is, the Cartan one-form $L^{\hat{r}}$ of (3.18). For its validity, we would like to note that it satisfies the two consistency conditions of the last section for the spin connection stemming from the symmetry superalgebra, which are $\hat{\omega}^{\hat{i}\hat{j}'} = 0$ and $\hat{\omega}^{-\hat{r}} = 0$ (just below of Eqs. (3.14) and (3.15)). Indeed, through the actual computation of the spin connection with the elfbein (4.1), we see that⁴

$$\hat{\omega}^{+I} = -\frac{1}{2} \partial_I A dx^- + \frac{1}{2} \delta^{I4} \left(\frac{\mu^2}{9} x^4 dx^- + \frac{\mu}{6} dx^9 \right), \quad \hat{\omega}^{+9} = \frac{\mu}{6} dx^4, \quad \hat{\omega}^{49} = -\frac{\mu}{6} dx^-, \quad (4.4)$$

and thus our parametrization is valid one.

Following the standard rule of KK reduction, we can directly read off from the elfbein (4.1) the ten dimensional quantities, that is, the zehnbein, the dilaton, and the Ramond-Ramond (R-R) one-form gauge field. In the string frame, the zehnbein is

$$e^+ = dx^+ - \frac{1}{2} A(x^I) dx^-, \quad e^- = dx^-, \quad e^I = dx^I, \quad (4.5)$$

from which the corresponding non-vanishing spin connection is obtained as

$$\omega^{+I} = -\frac{1}{2} \partial_I A dx^-. \quad (4.6)$$

The dilaton is trivially zero, $\phi = 0$, and the field strength of the R-R one-form gauge field is

$$F_{-4} = -\frac{\mu}{3}. \quad (4.7)$$

Together with the R-R four-form field strength $F_{-123} = \mu$ descending from the eleven dimensional plane wave background (3.1), (4.5) and (4.7) form the ten dimensional IIA plane wave background.

⁴We note that $\hat{\omega}^{49}$ is the type of $\hat{\omega}^{\hat{i}'\hat{j}'}$.

As shown in [22, 23] explicitly, the IIA plane wave background is not maximally supersymmetric and preserves 24 supersymmetries. To see this, let us consider the Killing spinor equation for our ten dimensional background,⁵

$$\Gamma^- \Gamma^4 (1 - \Gamma^{12349}) \epsilon = 0. \quad (4.8)$$

If we write the 32 component ϵ as $\epsilon = \epsilon^+ + \epsilon^-$ where $\epsilon^\pm \equiv \mathcal{P}_\pm \epsilon$ with the \mathcal{P}_\pm of (3.6), and introduce a new projection operator defined by

$$h_\pm \equiv \frac{1}{2}(1 \pm \Gamma^{12349}), \quad (4.9)$$

then it is easy to see that ϵ^+ and $h_+ \epsilon^-$ satisfy the Killing spinor equation and correspond to the supersymmetry of the IIA plane wave background. Because two projection operators commute with each other and each of them plays the role of filtering out half the components of ϵ , ϵ^+ and $h_+ \epsilon^-$ have 16 and 8 independent components respectively. This means that we get 24 supersymmetries in total. The remaining 8 components represented by $h_- \epsilon^-$ correspond to the broken supersymmetry. If we state this a little bit more, the projection operator

$$h_- \mathcal{P}_- \quad (4.10)$$

allows us to pick out the components of the spinorial quantity corresponding to the 8 broken supersymmetries.

4.2 Ten dimensional superfields

Now we turn to the KK reduction of eleven dimensional superfields. Similar to the previous bosonic case, there is a condition (the KK ansatz) that the super elfbein $L^{\hat{r}}$ should satisfy for the consistent KK reduction. It is $L_9^r = 0$ regarded as the superspace extension of (4.3) [42]. To check this condition, let us rewrite the eleven dimensional super-covariant derivative $\widehat{D}\theta$ appearing in $L^{\hat{r}}$ of (3.18) in terms of the ten dimensional quantities of the previous subsection. Then, from the expression of $\widehat{D}\theta$ given in (3.17), we have

$$\begin{aligned} \widehat{D}\theta &= \widehat{D}\theta^+ + \widehat{D}\theta^- \\ &= D\theta + \frac{\mu}{6} \Gamma^{-4} h_- \theta^- \hat{e}^9, \end{aligned} \quad (4.11)$$

⁵The Killing spinor equation is obtained from the supersymmetry variation of the dilatino field in the type IIA supergravity. Although there is another Killing spinor equation from the variation of the gravitino, it is not necessary in the current discussion. For more details, see [22] for example.

where h_- is the projection operator defined in (4.9) and $D\theta$ is the ten dimensional super-covariant derivative identified as

$$\begin{aligned} D\theta &= d\theta + \frac{1}{2}\omega^{+I}\Gamma_{+I}\theta + \frac{\mu}{12}(2e^i\Gamma^-\Pi\Gamma_i + e^{i'}\Gamma^-\Pi\Gamma_{i'} - 2e^4\Gamma^-\Pi\Gamma_4 h_-)\theta \\ &\quad - \frac{\mu}{12}e^-\Pi(\Gamma^-\Gamma^+ + 2h_-)\theta. \end{aligned} \quad (4.12)$$

From (4.11) and the expressions of \mathcal{M}^2 and \hat{e}^9 given in (3.19) and (4.1) respectively, it turns out that $L^{\hat{r}}$ in (3.18) has non-vanishing component in x^9 , especially, $L_9^r \neq 0$ except for $r = -$. Thus, the super elfbein does not satisfy the KK ansatz.

We would like to note that the obstacle for the dimensional reduction depends on $h_-\theta^-$ (or $h_-\mathcal{P}_-\theta$) which precisely corresponds to the components of eight broken supersymmetries. This is also the case in the construction of the superspace for the 24 supersymmetric $\text{AdS}_4 \times CP^3$ background through the dimensional reduction [13]. In some sense, this kind of structural similarity may be expected naturally because the IIA plane wave background is related to $\text{AdS}_4 \times CP^3$ via the Penrose limit. We may guess that, if some fraction of the supersymmetry was not broken along the direction of compactification, the super elfbein would not lead to any problem in going down to ten dimensions.

For the consistent KK reduction, the non-vanishing L_9^r component of the super elfbein should be eliminated. As has been done also in the case of $\text{AdS}_4 \times CP^3$ background [13], the way to eliminate it is to perform an appropriate local Lorentz transformation in the plane tangential to the eleven dimensional plane wave geometry. Then, let us denote the transformed super elfbein as $\hat{E}^{\hat{r}}$ and consider the Lorentz transformation,

$$\hat{E}^{\hat{r}} = L^{\hat{s}}\Lambda_{\hat{s}}^{\hat{r}}. \quad (4.13)$$

The problem is to determine the transformation matrix $\Lambda_{\hat{s}}^{\hat{r}}$ in such a way that the component \hat{E}_9^r of $\hat{E}^{\hat{r}}$ vanishes, that is, $\hat{E}_9^r = 0$. It is not so difficult to solve this. By the aid of the orthogonality condition,

$$\Lambda_{\hat{r}}^{\hat{t}}\Lambda_{\hat{s}}^{\hat{u}}\eta_{\hat{t}\hat{u}} = \eta_{\hat{r}\hat{s}}, \quad (4.14)$$

and the requirement of proper Lorentz transformation, $\det \Lambda_{\hat{r}}^{\hat{s}} = +1$, $\Lambda_{\hat{s}}^{\hat{r}}$ is uniquely determined as

$$\begin{aligned} \Lambda_9^9 &= \frac{1}{\sqrt{1+v^2}}, \quad \Lambda_r^9 = \frac{\eta_{rs}v^s}{\sqrt{1+v^2}}, \quad \Lambda_9^r = -\frac{v^r}{\sqrt{1+v^2}}, \\ \Lambda_s^r &= \delta_s^r - \frac{\sqrt{1+v^2}-1}{v^2\sqrt{1+v^2}}\eta_{st}v^tv^r, \end{aligned} \quad (4.15)$$

where we have defined

$$v^r \equiv \frac{L_9^r}{L_9^9}, \quad v^2 \equiv \eta_{rs} v^r v^s. \quad (4.16)$$

Here $L_9^r = 0$ and $L_9^9 = 1$ ($v^r = 0$) when $h_- \theta^- = 0$. Thus the transformation matrix $\Lambda_{\hat{s}}^{\hat{r}}$ becomes the unit matrix when the fermionic components corresponding to the broken supersymmetry vanish. One property of v^r is that v^- always vanishes because $L_9^- = 0$ as mentioned below of (4.12), and thus $v^2 = v^I v^I$.

The Lorentz transformation (4.13) with the transformation matrix (4.15) is the one for the vector quantities. In addition to this, we should perform a corresponding Lorentz transformation also for the spinor superfield L ,

$$\widehat{E}^{\hat{a}} = L^{\hat{b}} S_b^{\hat{a}}. \quad (4.17)$$

The transformation matrix S is derived by making use of the relation between the vector and the spinor representations of the Lorentz group,

$$S^{T-1} \Gamma^{\hat{r}} S^T = \Gamma^{\hat{s}} \Lambda_{\hat{s}}^{\hat{r}}. \quad (4.18)$$

If we take the standard expression $S^T = \exp\left(\frac{1}{4} \Gamma_{\hat{r}\hat{s}} \epsilon^{\hat{r}\hat{s}}\right)$ with the transformation parameter $\epsilon^{\hat{r}\hat{s}}$ and investigate the infinitesimal transformation, it follows that Γ_{r9} generates the Lorentz transformation for the spinorial quantities. Based on this, we can obtain the explicit expression of S for finite transformation as

$$\begin{aligned} S &= \exp\left(\frac{1}{2} \Gamma_{r9}^T \epsilon^{r9}\right) \quad \text{with} \quad \epsilon^{r9} = -\tan^{-1} |v| \frac{v^r}{|v|} \\ &= \frac{1}{\sqrt{2}(1+v^2)^{1/4}} \left(\mathbf{1}_{32} \sqrt{\sqrt{1+v^2}+1} - \Gamma_{r9}^T \frac{v^r}{|v|} \sqrt{\sqrt{1+v^2}-1} \right), \end{aligned} \quad (4.19)$$

where $\mathbf{1}_{32}$ is the 32×32 unit matrix.

The transformed super elfbein, $\widehat{E}^{\hat{r}}$ and $\widehat{E}^{\hat{a}}$, has the required form for the KK reduction. Thus, we are now ready to get the ten dimensional superfields by following the relation between the eleven and ten dimensional quantities [42]. First of all, the dilaton and dilatino superfield are obtained as

$$\begin{aligned} \Phi^{2/3} &= \widehat{E}_9^9 = L_9^9 \sqrt{1+v^2}, \\ \chi^a &= \Phi^{1/3} \widehat{E}_9^a = \Phi^{1/3} L_9^b S_b^a. \end{aligned} \quad (4.20)$$

We note that, when the eleven dimensional spinorial quantity is related to the ten dimensional one, a dilaton factor $e^{\phi/6}$ should be multiplied for each spinor index such as $\theta^a = e^{\phi/6} \theta^{\hat{a}}$. This is basically due to the necessity for having the canonical supersymmetry

transformation rule in ten dimensions. However, the dilaton field is trivial in the IIA plane wave background, and thus the dilaton factor does not appear in the above expression.

As for the super zehnbein, we get

$$\begin{aligned}
E^r &= \Phi^{1/3} dZ^M \widehat{E}_M^r \\
&= dZ^M (\Phi^{1/3} L_M^s \Lambda_s^r - \Phi^{-1/3} L_M^9 L_9^r), \\
E^a &= dZ^M (\Phi^{1/3} \widehat{E}_M^a - \Phi^{-1/3} \widehat{E}_M^9 \widehat{E}_9^a) \\
&= dZ^M \left(\Phi^{1/3} L_M^b - \frac{\Phi^{-1/3}}{\sqrt{1+v^2}} (L_M^r v_r + L_M^9) L_9^b \right) S_b^a,
\end{aligned} \tag{4.21}$$

where Λ_s^r and S_b^a are the Lorentz transformation matrices of (4.15) and (4.19). We note that, although it is clear from the form $dZ^M L_M^A$, the ten dimensional super-covariant derivative (4.12) is used in super elfbein instead of the eleven dimensional one (4.11). That is, we use the super elfbein given in (3.18) but with the replacement of $\widehat{D}\theta$ by $D\theta$. One may think that \widehat{e}^9 appearing in L_M^9 and the relation between the eleven and ten dimensional super-covariant derivatives (4.11) somehow contributes to super zehnbein because we have $\widehat{e}_-^9 \neq 0$ from (4.1) which corresponds to the non-vanishing R-R one-form gauge field. However, an explicit calculation shows that such part does not contribute to the super zehnbein. In fact, this should be the case because the super zehnbein is neutral under the gauge transformation associated with the R-R one-form gauge field. Thus, it is understood that, in the actual evaluation of (4.21), \widehat{e}_μ^9 is ignored and the super-covariant derivative is ten dimensional one.

We turn to the three-form superfield \widehat{B} . As one can see from its expression (3.23), it is a Lorentz scalar because it does not contain any index in tangent space. Therefore, the effect of the local Lorentz transformation is just to replace the quantities in its expression with the transformed ones, and what we get after the transformation is

$$\widehat{B} = \frac{1}{6} \widehat{e}^{\hat{r}} \wedge \widehat{e}^{\hat{s}} \wedge \widehat{e}^{\hat{t}} \widehat{C}_{\hat{r}\hat{s}\hat{t}} + \int_0^1 d\lambda \bar{\theta}' \Gamma_{\hat{r}\hat{s}} \widehat{E}_\lambda \wedge \widehat{E}_\lambda^{\hat{r}} \wedge \widehat{E}_\lambda^{\hat{s}}, \tag{4.22}$$

where the subscript λ means the rescaling $\theta \rightarrow \lambda\theta$ in the superfields as in (3.23) and $\bar{\theta}'$ is the transformed fermionic coordinate given by⁶

$$\bar{\theta}' = \bar{\theta} (S_\lambda^T)^{-1}, \tag{4.23}$$

with $S_\lambda^T = S^T|_{\theta \rightarrow \lambda\theta}$. The first term on the right hand side remains intact under the Lorentz transformation because it is originated from the purely bosonic part of the closed four-form

⁶We have used $SCS^T = C$, the property of the charge conjugation matrix C under the Lorentz transformation.

H , $H_{\lambda=0}$, as can be seen from (3.21), and the effect of Lorentz transformation with the transformation matrix $\Lambda_{\hat{s}}^{\hat{r}}$ disappears at $\lambda = 0$.⁷

Under the KK reduction, the three-form superfield gives R-R three-form and NS-NS two-form gauge superfields in ten dimensions. Here, we restrict our attention to the NS-NS superfield because it is relevant in constructing the superstring action. If we denote it as $B = \frac{1}{2}dZ^M \wedge dZ^N B_{NM}$, then B_{NM} corresponds to \widehat{B}_{9NM} [42] and B is given by

$$\begin{aligned} B &= \frac{1}{2}dZ^M \wedge dZ^N \widehat{B}_{9NM} \\ &= \int_0^1 d\lambda \left(2\bar{\theta}'\Gamma_{r9}\widehat{E}_\lambda \wedge \widehat{E}_\lambda^r \widehat{E}_{\lambda 9}^9 + 2\bar{\theta}'\Gamma_{r9}\widehat{E}_{\lambda 9}\widehat{E}_\lambda^r \wedge \widehat{E}_\lambda^9 + \bar{\theta}'\Gamma_{rs}\widehat{E}_{\lambda 9}\widehat{E}_\lambda^r \wedge \widehat{E}_\lambda^s \right) \\ &= \int_0^1 d\lambda \left(2\bar{\theta}'\Gamma_{r9}E_\lambda \wedge E_\lambda^r + \Phi_\lambda^{-1}\bar{\theta}'\Gamma_{rs}\chi_\lambda E_\lambda^r \wedge E_\lambda^s \right), \end{aligned} \quad (4.24)$$

where the three-form gauge field $\widehat{C}_{\hat{r}\hat{s}\hat{t}}$ does not contribute to B because its field strength does not span along x^9 direction in the plane wave background (3.1).

5 Type IIA superstring action on IIA plane wave background

We have obtained all the ten dimensional superfields necessary for the construction of type IIA superstring action on the IIA plane wave background, and are now ready to write down the action containing all the 32 fermionic components.

The general form of type IIA Green-Schwarz superstring action is

$$S_{\text{IIA}} = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} h^{mn} \Pi_m^r \Pi_n^s \eta_{rs} + \frac{1}{2\pi\alpha'} \int B_2, \quad (5.1)$$

where Π_m^A and B_2 are the pullback of the super zehnbein and the NS-NS two-form gauge superfield onto the string worldsheet given by

$$\begin{aligned} \Pi_m^A &= \partial_m Z^M E_M^A, \\ B_2 &= \frac{1}{2}d^2\sigma \epsilon^{mn} \partial_m Z^M \partial_n Z^N B_{NM}. \end{aligned} \quad (5.2)$$

As for the worldsheet quantities, σ^m ($m = 0, 1$) is the worldsheet coordinate with the usual notation

$$\sigma^0 = \tau, \quad \sigma^1 = \sigma, \quad (5.3)$$

⁷Here, $\Lambda_{\hat{s}}^{\hat{r}}$ is also understood as the rescaled one through $\theta \rightarrow \lambda\theta$.

h^{mn} is the worldsheet metric, and the anti-symmetric tensor ϵ^{mn} follows the convention $\epsilon^{01} = +1$.

If we now plug the expression of (4.24) for the NS-NS two-form superfield into the superstring action, then we have

$$S_{\text{IIA}} = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} h^{mn} \partial_m Z^M E_M^r \partial_n Z^N E_N^s \eta_{rs} \\ + \frac{1}{2\pi\alpha'} \int d^2\sigma \int_0^1 d\lambda \left(2\bar{\theta}' \Gamma_{r9} E_\lambda \wedge E_\lambda^r + \Phi_\lambda^{-1} \bar{\theta}' \Gamma_{rs} \chi_\lambda E_\lambda^r \wedge E_\lambda^s \right), \quad (5.4)$$

where the expressions for various superfields are given in (4.20) and (4.21). Thus, we have achieved our goal of constructing the complete type IIA Green-Schwarz superstring action on the IIA plane wave background. However, the action depends implicitly on other expressions such as the super elfbein of (3.18) and the Lorentz transformation matrices in (4.15) and (4.19). To facilitate the better understanding of the derived results, we perform a little bit more manipulation for the superstring action and give a summary of related expressions.

The superstring action is composed of the kinetic and the Wess-Zumino term:

$$S_{\text{IIA}} = S_{\text{kin}} + S_{\text{WZ}}. \quad (5.5)$$

Let us first consider the kinetic term. Then (4.20) and (4.21) allow us to express it as follows.

$$S_{\text{kin}} = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} h^{mn} \partial_m Z^M \partial_n Z^N (-1)^{\langle M, N \rangle} (\Phi^{2/3} L_M^r L_N^s \eta_{rs} \\ - \Phi^{-2/3} L_M^r L_N^s L_9^t \eta_{rt} \eta_{su} - 2\Phi^{-2/3} L_M^r L_N^9 L_9^s \eta_{rs} + \Phi^{-2/3} L_M^9 L_N^9 L_9^r \eta_{rs}), \quad (5.6)$$

where the expressions of (4.15) for the Lorentz transformation, and the properties of the Lorentz transformation matrices, (4.14) and (4.18), have been used. The symbol $\langle M, N \rangle$ means that $\langle M, N \rangle = 1$ when both of M and N are spinorial and $\langle M, N \rangle = 0$ otherwise. Similar manipulation for the Wess-Zumino term leads us to have

$$S_{\text{WZ}} = \frac{1}{2\pi\alpha'} \int d^2\sigma \int_0^1 d\lambda \left(\bar{\theta} \Gamma_{rs} L_{\lambda 9} L_\lambda^r \wedge L_\lambda^s + 2\bar{\theta} \Gamma_{r9} L_\lambda \wedge L_\lambda^r L_{\lambda 9}^9 + 2\bar{\theta} \Gamma_{rs} L_\lambda \wedge L_\lambda^r L_{\lambda 9}^s \right. \\ \left. + 2\bar{\theta} \Gamma_{r9} L_{\lambda 9} L_\lambda^r \wedge L_\lambda^9 - 2\bar{\theta} \Gamma_{r9} L_\lambda \wedge L_\lambda^9 L_{\lambda 9}^r \right), \quad (5.7)$$

where the wedge product is understood as the pullback version, that is, for example

$$L^r \wedge L^s \equiv \epsilon^{mn} \partial_m Z^M L_M^r \partial_n Z^N L_N^s. \quad (5.8)$$

Explicit expressions for the various quantities appearing in the action are given by

$$\Phi^{2/3} = \sqrt{(L_9^9)^2 + L_9^r L_9^s \eta_{rs}},$$

$$\begin{aligned}
dZ^M L_M^r &= e^r - 2 \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta} \Gamma^r \mathcal{M}^{2n} D\theta, \\
dZ^M L_M^9 &= -2 \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta} \Gamma^9 \mathcal{M}^{2n} D\theta, \\
dZ^M L_M &= \sum_{n=0}^{16} \frac{1}{(2n+1)!} \mathcal{M}^{2n} D\theta, \\
L_9^r &= -\frac{\mu}{3} \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta} \Gamma^r \mathcal{M}^{2n} \Gamma^{-4} h_- \theta^-, \\
L_9^9 &= 1 - \frac{\mu}{3} \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta} \Gamma^9 \mathcal{M}^{2n} \Gamma^{-4} h_- \theta^-, \\
L_9 &= \frac{\mu}{6} \sum_{n=0}^{16} \frac{1}{(2n+1)!} \mathcal{M}^{2n} \Gamma^{-4} h_- \theta^-,
\end{aligned} \tag{5.9}$$

where the ten dimensional super-covariant one form $D\theta$ and the 32×32 matrix \mathcal{M}^2 are given in (4.12) and (3.19) respectively.

5.1 Light-cone gauge fixed action

The superstring action (5.5) is a complete action containing all the 32 fermionic coordinates. We now take the fermionic and the bosonic light-cone gauge choices and obtain the superstring action in the light-cone gauge.

We first fix the fermionic κ -symmetry by taking the usual κ -symmetry light cone gauge

$$\Gamma^- \theta = 0 \quad (\theta^- = \mathcal{P}_- \theta = 0). \tag{5.10}$$

Under this choice, it follows immediately that

$$L_9^r = 0, \quad L_9^9 = 1, \quad L_9 = 0, \quad \Phi = 1, \tag{5.11}$$

as can be seen from (5.9). The super-covariant one form $D\theta$ of (4.12) is simplified as

$$D\theta = d\theta - \frac{\mu}{4} e^- \left(\Gamma^{123} + \frac{1}{3} \Gamma^{49} \right) \theta. \tag{5.12}$$

As for the matrix \mathcal{M}^2 of (3.19), it simply vanishes due to the presence of projection operator \mathcal{P}_- in every terms. This fact leads to a pretty much simplification for the remaining superfields of (5.9) as follows.

$$dZ^M L_M^r = e^r - \bar{\theta} \Gamma^r D\theta, \quad dZ^M L_M^9 = -\bar{\theta} \Gamma^9 D\theta, \quad dZ^M L_M = D\theta, \tag{5.13}$$

with $D\theta$ of (5.12).

If we plug the above expressions from (5.11) to (5.13) into the superstring action (5.5) and use the bosonic zweibein of (4.5), then we obtain the κ -symmetry fixed superstring action as

$$\begin{aligned}
S_{\text{IIA}} = & -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} h^{mn} \left[2\partial_m X^+ \partial_n X^- + \partial_m X^I \partial_n X^I - A(X^I) \partial_m X^- \partial_n X^- \right. \\
& + 2\partial_m X^- \bar{\theta} \Gamma^+ \partial_n \theta + \frac{\mu}{2} \partial_m X^- \partial_n X^- \bar{\theta} \Gamma^+ \left(\Gamma^{123} + \frac{1}{3} \Gamma^{49} \right) \theta \left. \right] \\
& - \frac{1}{2\pi\alpha'} \int d^2\sigma \epsilon^{mn} \partial_m X^- \bar{\theta} \Gamma^{+9} \partial_n \theta .
\end{aligned} \tag{5.14}$$

We now turn to the bosonic light-cone gauge. The equation of motion for X^- is harmonic, and thus the light-cone gauge, $X^- \propto \tau$, is allowed. Let us take the following light-cone gauge choice

$$X^- = \alpha' p^- \tau, \tag{5.15}$$

where p^- is the total momentum conjugate to X^+ . With this choice, the worldsheet diffeomorphism can be consistently fixed as $\sqrt{-h} = 1$, $h_{\sigma\tau} = 0$, which allow us to fix other worldsheet metric components as $h_{\tau\tau} = -1$ and $h_{\sigma\sigma} = 1$. Then the κ -symmetry fixed superstring action (5.14) is further simplified, and the superstring action in the light-cone gauge, S_{LC} , is obtained finally as

$$\begin{aligned}
S_{\text{LC}} = & -\frac{1}{4\pi\alpha'} \int d^2\sigma \left[\eta^{mn} \partial_m X^I \partial_n X^I + \frac{m^2}{9} (X^i)^2 + \frac{m^2}{36} (X^{i'})^2 \right. \\
& \left. - \bar{\theta} \Gamma^+ \partial_\tau \theta + \bar{\theta} \Gamma^{+9} \partial_\sigma \theta - \frac{m}{4} \bar{\theta} \Gamma^+ \left(\Gamma^{123} + \frac{1}{3} \Gamma^{49} \right) \theta \right],
\end{aligned} \tag{5.16}$$

where the fermionic coordinate has been rescaled as $\theta \rightarrow \theta/\sqrt{2\alpha'p^-}$ and m defined as $m \equiv \mu\alpha'p^-$ is a mass parameter for the worldsheet variables. This is the action considered previously [21, 22], and thus shows that our complete superstring action of (5.5) satisfies a basic consistency check.

6 Conclusion

We have constructed the complete type IIA Green-Schwarz superstring action on the ten dimensional IIA plane wave background with 24 supersymmetries. As a consistency check, we have obtained the superstring action in the light-cone gauge and shown that it is exactly the same as that considered previously.

Having the complete action containing all the 32 fermionic components, we can study the various possible superstring configurations by taking an appropriate κ -symmetry fixing condition. Especially interesting thing is the configuration whose correct quantum description requires the fermionic components corresponding to the broken supersymmetries. As noted in the introduction, in the case of type IIA superstring on $\text{AdS}_4 \times CP^3$, such fermionic components are crucial for the description of superstring moving only in AdS_4 space [13]. Under the Penrose limit relating the $\text{AdS}_4 \times CP^3$ space to the type IIA plane wave background, the string configuration embedded only in AdS_4 space would correspond to that spanned in the space parametrized by x^i . Since the superstring on the type IIA plane wave background is rather simpler than that on $\text{AdS}_4 \times CP^3$ space, we expect that we can understand such superstring configuration more clearly.

Another interesting issue that may be considered with the complete superstring action is the realization of worldsheet supersymmetry. As has been shown in [22], the superstring action in the light-cone gauge (5.16) has $\mathcal{N} = (4, 4)$ worldsheet supersymmetry. Among the 16 fermionic components eliminated by the κ -symmetry light cone gauge $\Gamma^- \theta = 0$, 8 components correspond to the broken supersymmetries. Other 8 components are responsible for the worldsheet supersymmetry which basically stems from the fact that the supersymmetry transformation parameter $h_+ \epsilon^-$ discussed at the end of Sec. 4.1 does not satisfy the κ -symmetry light-cone gauge. At this point, one may be curious about the worldsheet supersymmetry realized after taking another consistent κ -symmetry fixing condition. In the case of maximally supersymmetric backgrounds, there would be nothing special and we would get unique structure on worldsheet supersymmetry. However, it seems that there would be some change in the supersymmetry structure in less supersymmetric backgrounds such as the present IIA plane wave background. The work on this issue is in progress, and will be reported elsewhere.

Acknowledgments

HS would like to thank Makoto Sakaguchi for helpful discussion and informing his work relevant to this work. This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) with the Grants No. 2009-0085995 (JP), 2008-331-C00071 (HS) and 2005-0049409 (JP) through the Center for Quantum Space-time (CQUeST) of Sogang University. JP is also supported by POSTECH BSRI fund with the Grants No. 4.0007999.01 and appreciates APCTP for its stimulating environment for research.

A Notation

The eleven dimensional quantities and indices are denoted basically by using hat to distinguish from those in ten dimensions.

In eleven dimensions, $\hat{M}, \hat{N}, \dots (\hat{A}, \hat{B}, \dots)$ are the target (tangent) superspace indices. Each superspace index is the composition of two types of indices such as $\hat{M} = (\hat{\mu}, \hat{\alpha})$ ($\hat{A} = (\hat{r}, \hat{a})$). Convention for each index and related indices is as follows:

$\hat{r}, \hat{s}, \dots = +, -, 1, \dots, 9$	11D tangent space-time vector indices
$\hat{\mu}, \hat{\nu}, \dots = +, -, 1, \dots, 9$	11D target space-time vector indices
$\hat{a}, \hat{b}, \dots = 1, \dots, 32$	11D tangent space-time spinor indices
$\hat{\alpha}, \hat{\beta}, \dots = 1, \dots, 32$	11D target space-time spinor indices
$\hat{I}, \hat{J}, \dots = 1, \dots, 9$	$SO(9)$ vector indices ($\hat{I} = (\hat{i}, \hat{i}')$)
$\hat{i}, \hat{j}, \dots = 1, 2, 3$	
$\hat{i}', \hat{j}', \dots = 4, \dots, 9$	

The metric $\eta_{\hat{r}\hat{s}}$ for the tangent space-time follows the most plus convention. Since the light-cone coordinate is defined as

$$x^{\pm} \equiv \frac{1}{\sqrt{2}}(x^{11} \pm x^0), \quad (\text{A.1})$$

$\eta_{+-} = 1$ with the spatial part $\eta_{\hat{I}\hat{J}} = \delta_{\hat{I}\hat{J}}$.

In ten dimensions, $M, N, \dots (A, B, \dots)$ are the target (tangent) superspace indices. Similar to the eleven dimensional case, $M = (\mu, \alpha)$ ($A = (r, a)$) with the following convention.

$r, s, \dots = +, -, 1, \dots, 8$	10D tangent space-time vector indices
$\mu, \nu, \dots = +, -, 1, \dots, 8$	10D target space-time vector indices
$a, b, \dots = 1, \dots, 32$	10D tangent space-time spinor indices
$\hat{\alpha}, \hat{\beta}, \dots = 1, \dots, 32$	10D target space-time spinor indices
$I, J, \dots = 1, \dots, 8$	$SO(8)$ vector indices ($I = (i, i')$)
$i, j, \dots = 1, 2, 3, 4$	
$i', j', \dots = 5, 6, 7, 8$	

References

- [1] J. H. Schwarz, “Superconformal Chern-Simons theories,” JHEP **0411** (2004) 078 [arXiv:hep-th/0411077].
- [2] J. Bagger and N. Lambert, “Modeling multiple M2’s,” Phys. Rev. D **75** (2007) 045020 [arXiv:hep-th/0611108].
- [3] J. Bagger and N. Lambert, “Gauge symmetry and supersymmetry of multiple M2-branes,” Phys. Rev. D **77** (2008) 065008 [arXiv:0711.0955 [hep-th]].
- [4] J. Bagger and N. Lambert, “Comments on multiple M2-branes,” JHEP **0802** (2008) 105 [arXiv:0712.3738 [hep-th]].
- [5] A. Gustavsson, “Algebraic structures on parallel M2-branes,” Nucl. Phys. B **811**, 66 (2009) [arXiv:0709.1260 [hep-th]].
- [6] A. Gustavsson, “Selfdual strings and loop space Nahm equations,” JHEP **0804**, 083 (2008) [arXiv:0802.3456 [hep-th]].
- [7] M. Van Raamsdonk, “Comments on the Bagger-Lambert theory and multiple M2-branes,” JHEP **0805**, 105 (2008) [arXiv:0803.3803 [hep-th]].
- [8] D. Gaiotto and E. Witten, “Janus configurations, Chern-Simons couplings, and the theta-angle in N=4 Super Yang-Mills theory,” JHEP **1006** (2010) 097 [arXiv:0804.2907 [hep-th]].
- [9] K. Hosomichi, K. M. Lee, S. Lee, S. Lee and J. Park, “N=4 Superconformal Chern-Simons theories with hyper and twisted hyper multiplets,” JHEP **0807**, 091 (2008) [arXiv:0805.3662 [hep-th]].
- [10] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, “N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals,” JHEP **0810**, 091 (2008) [arXiv:0806.1218 [hep-th]].
- [11] K. Hosomichi, K. Lee, S. Lee, S. Lee and J. Park, “N=5,6 Superconformal Chern-Simons Theories and M2-branes on Orbifolds,” JHEP **0809** 002 (2008) [arXiv:0806.4977 [hep-th]].
- [12] G. Arutyunov and S. Frolov, “Superstrings on $AdS(4) \times CP^{*3}$ as a Coset Sigma-model,” JHEP **0809** (2008) 129 [arXiv:0806.4940 [hep-th]];

- B. Stefanski jr, “Green-Schwarz action for Type IIA strings on $AdS_4 \times CP^3$,” Nucl. Phys. B **808** (2009) 80 [arXiv:0806.4948 [hep-th]].
- [13] J. Gomis, D. Sorokin and L. Wulff, “The Complete $AdS(4) \times CP^{**3}$ superspace for the type IIA superstring and D-branes,” JHEP **0903** (2009) 015 [arXiv:0811.1566 [hep-th]].
- [14] M. Blau, J. M. Figueroa-O’Farrill, C. Hull and G. Papadopoulos, “A New maximally supersymmetric background of IIB superstring theory,” JHEP **0201** (2002) 047 [hep-th/0110242].
- [15] M. Blau, J. M. Figueroa-O’Farrill, C. Hull and G. Papadopoulos, “Penrose limits and maximal supersymmetry,” Class. Quant. Grav. **19** (2002) L87 [hep-th/0201081];
M. Blau, J. M. Figueroa-O’Farrill and G. Papadopoulos, “Penrose limits, supergravity and brane dynamics,” Class. Quant. Grav. **19** (2002) 4753 [hep-th/0202111].
- [16] R. R. Metsaev, “Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background,” Nucl. Phys. B **625** (2002) 70 [hep-th/0112044].
- [17] D. Berenstein, J. M. Maldacena, and H. Nastase, “Strings in flat space and pp waves from $N = 4$ super Yang Mills,” JHEP **04** (2002) 013, hep-th/0202021.
- [18] T. Nishioka and T. Takayanagi, “On Type IIA Penrose Limit and $N=6$ Chern-Simons Theories,” JHEP **0808** (2008) 001 [arXiv:0806.3391 [hep-th]].
- [19] D. Gaiotto, S. Giombi and X. Yin, “Spin Chains in $N=6$ Superconformal Chern-Simons-Matter Theory,” JHEP **0904** (2009) 066 [arXiv:0806.4589 [hep-th]].
- [20] G. Grignani, T. Harmark and M. Orselli, “The $SU(2) \times SU(2)$ sector in the string dual of $N=6$ superconformal Chern-Simons theory,” Nucl. Phys. B **810** (2009) 115 [arXiv:0806.4959 [hep-th]].
- [21] K. Sugiyama and K. Yoshida, “Type IIA string and matrix string on PP wave,” Nucl. Phys. B **644** (2002) 128 [hep-th/0208029].
- [22] S. j. Hyun and H. j. Shin, “ $N=(4,4)$ type IIA string theory on PP wave background,” JHEP **0210** (2002) 070 [arXiv:hep-th/0208074].
- [23] I. Bena and R. Roiban, “Supergravity pp wave solutions with twenty eight supercharges and twenty four supercharges,” Phys. Rev. D **67** (2003) 125014 [hep-th/0206195].

- [24] S. -j. Hyun and H. -j. Shin, “Solvable $N=(4,4)$ type IIA string theory in plane wave background and D-branes,” Nucl. Phys. B **654** (2003) 114 [hep-th/0210158].
- [25] D. Astolfi, V. G. M. Puletti, G. Grignani, T. Harmark and M. Orselli, “Finite-size corrections in the $SU(2) \times SU(2)$ sector of type IIA string theory on $AdS(4) \times CP^{**3}$,” Nucl. Phys. B **810** (2009) 150 [arXiv:0807.1527 [hep-th]].
- [26] D. Astolfi, V. G. M. Puletti, G. Grignani, T. Harmark and M. Orselli, “Full Lagrangian and Hamiltonian for quantum strings on $AdS(4) \times CP^{**3}$ in a near plane wave limit,” JHEP **1004** (2010) 079 [arXiv:0912.2257 [hep-th]].
- [27] A. Agarwal and D. Young, “ $SU(2|2)$ for Theories with Sixteen Supercharges at Weak and Strong Coupling,” Phys. Rev. D **82** (2010) 045024 [arXiv:1003.5547 [hep-th]].
- [28] M. Ali-Akbari, “A D2-brane in the Penrose limits of $AdS(4) \times CP(3)$,” Phys. Rev. D **82** (2010) 065027 [arXiv:1005.0126 [hep-th]].
- [29] D. Astolfi, V. G. M. Puletti, G. Grignani, T. Harmark and M. Orselli, “Finite-size corrections for quantum strings on $AdS_4 \times CP^3$,” JHEP **1105** (2011) 128 [arXiv:1101.0004 [hep-th]].
- [30] D. Astolfi, G. Grignani, E. Ser-Giacomi and A. V. Zayakin, “Strings in $AdS_4 \times CP^3$: finite size spectrum vs. Bethe Ansatz,” arXiv:1111.6628 [hep-th].
- [31] P. A. Grassi, D. Sorokin and L. Wulff, “Simplifying superstring and D-brane actions in $AdS_4 \times CP^3$ superbackground,” JHEP **0908** (2009) 060 [arXiv:0903.5407 [hep-th]].
- [32] B. de Wit, K. Peeters, J. Plefka, A. Sevrin, “The M theory two-brane in $AdS(4) \times S^{**7}$ and $AdS(7) \times S^{**4}$,” Phys. Lett. **B443** (1998) 153-158. [hep-th/9808052].
- [33] R. Kallosh, J. Rahmfeld and A. Rajaraman, “Near horizon superspace,” JHEP **9809** (1998) 002 [arXiv:hep-th/9805217].
- [34] J. Kowalski-Glikman, “Vacuum States in Supersymmetric Kaluza-Klein Theory,” Phys. Lett. B **134** (1984) 194.
- [35] J. M. Figueroa-O’Farrill and G. Papadopoulos, “Maximally supersymmetric solutions of ten-dimensional and eleven- dimensional supergravities,” JHEP **0303** (2003) 048 [hep-th/0211089].

- [36] J. M. Figueroa-O'Farrill and G. Papadopoulos, "Homogeneous fluxes, branes and a maximally supersymmetric solution of M theory," JHEP **0108** (2001) 036 [hep-th/0105308];
- [37] M. Hatsuda, K. Kamimura and M. Sakaguchi, "Super-PP wave algebra from super-AdS x S algebras in eleven-dimensions," Nucl. Phys. B **637** (2002) 168 [arXiv:hep-th/0204002].
- [38] R. R. Metsaev and A. A. Tseytlin, "Type IIB superstring action in AdS(5) x S**5 background," Nucl. Phys. B **533** (1998) 109 [hep-th/9805028].
- [39] M. Sakaguchi and K. Yoshida, "Non-relativistic AdS branes and Newton-Hooke super-algebra," JHEP **0610** (2006) 078 [arXiv:hep-th/0605124].
- [40] J. A. De Azcarraga and P. K. Townsend, "Superspace Geometry And Classification Of Supersymmetric Extended Objects," Phys. Rev. Lett. **62** (1989) 2579.
- [41] J. Michelson, "Twisted toroidal compactification of pp waves," Phys. Rev. D **66** (2002) 066002 [hep-th/0203140].
- [42] M. J. Duff, P. S. Howe, T. Inami and K. S. Stelle, "Superstrings in D=10 from Super-membranes in D=11," Phys. Lett. B **191** (1987) 70.